73050 Tilastomatematiikka **Tentti 20.8.2001**

Ei laskinta eikä kirjallisuutta, kaavakokoelma jaetaan

1. Millä todennäköisyydellä väliltä 1 - 100 satunnaisesti valittu kokonaisluku on jaollinen luvuilla 3 tai 5 ? Käytä kaavakokoelman kaavaa 1.

2. Määritä $P(x > 1)$ Poisson-jakaumassa, jossa $P(x = 0) = P(x > 0)$

3. Diskreetti satunnaisvektori (x, y) on tasaisesti jakautunut otosavaruuteen $\{(1,3),(1,4),(2,3),(2,4)\}$.

a) Laske $\mathrm{cov}(x, y)$. Ovatko x ja y riippumattomia?

b) Olkoon $u = x + y$, $v = x - y$.
Laske $\mathrm{cov}(u, v)$. Ovatko u ja v riippumattomia?

4. Kahta vaakaa käyttäen saadaan mittaustulokset

$$x_1 = m + \varepsilon_1 \text{ , missä } \varepsilon_1 \sim N(0,2)$$
$$x_2 = m + \varepsilon_2 \text{ , missä } \varepsilon_2 \sim N(0,5)$$

Antaako tarkimman tuloksen mittaus 1, mittaus 2 vai näiden keskiarvo? Keksitkö vielä paremman tavan käyttää tuloksia x_1 ja x_2?

5. Tutkittiin muuttujan x vaikutusta suureeseen y. Saatiin koetulokset :

x:	3	4	5	6
y:	2	1	3	7

a) Estimoi regressiomallin $y = \beta_1 + \beta_2(x-4)^2 + u$ kertoimet β_1 ja β_2.

(Ole huolellinen, kun muodostat matriisia X . Silloin tehtävä onnistuu ilman laskintakin.)

b) Testaa hypoteesi $H_0: \beta_2 = 0$ vaihtoehtoista hypoteesia $H_1: \beta_2 \neq 0$ vastaan merkitsevyystasoilla 5%, 1% ja 0.1%.

73050 Tilastomatematiikan kaavoja ja taulukoita

1. $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j)$
$+ \sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$

2. $P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2)\cdots P\left(A_n \mid \bigcap_{i=1}^{n-1} A_i\right)$

3. $P(B_k \mid A) = \dfrac{P(B_k)P(A \mid B_k)}{\sum_{i=1}^{n} P(B_i)P(A \mid B_i)}$

4. $\hat{f}(y) = f\left(h^{-1}(y)\right)\left|\dfrac{d}{dy} h^{-1}(y)\right|$

5. $\mathrm{var}(x) = E\left(x^2\right) - [E(x)]^2$

6. $P(|x - \mu| \ge t) \le \dfrac{\sigma^2}{t^2} \quad \forall t > 0$

7. $b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$, $p(x; \mu) = \dfrac{\mu^x}{x!} e^{-\mu}$

8. $\mathrm{cov}(x, y) = E(x\,y) - E(x)E(y)$

9. $\mathrm{var}(a\,x + b\,y) = a^2\,\mathrm{var}(x) + b^2\,\mathrm{var}(y) + 2ab\,\mathrm{cov}(x, y)$

10. $\bar{x} \sim N\left(\mu, \sigma^2 / n\right)$

11. $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$

12. $\dfrac{(\bar{x} - \mu)}{s / \sqrt{n}} \sim t(n-1)$

13. $\dfrac{s_x^2 / \sigma_x^2}{s_y^2 / \sigma_y^2} \sim F\left(n_x - 1, n_y - 1\right)$

14. $\left[\bar{x} - z_1 \dfrac{\sigma}{\sqrt{n}},\ \bar{x} + z_1 \dfrac{\sigma}{\sqrt{n}}\right]$; $P(-z_1 \le z \le z_1) = 1 - \alpha$

15. $\left[\bar{x} - t_1 \dfrac{s}{\sqrt{n}},\ \bar{x} + t_1 \dfrac{s}{\sqrt{n}}\right]$; $P(-t_1 \le t \le t_1) = 1 - \alpha$

16. $\left[\dfrac{(n-1)s^2}{w_2},\ \dfrac{(n-1)s^2}{w_1}\right]$; $P(h < w_1) = P(h > w_2) = \dfrac{\alpha}{2}$

17. $\left[\hat{p} - z_1\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_1\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right]$; $P(-z_1 \le z \le z_1) = 1 - \alpha$

18. $\dfrac{\bar{x} - \bar{y} - \left(\mu_x - \mu_y\right)}{\sqrt{s_p^2\left(1/n_x + 1/n_y\right)}} \sim t\left(n_x + n_y - 2\right)$; $s_p^2 = \dfrac{(n_x - 1)s_x^2 + \left(n_y - 1\right)s_y^2}{n_x + n_y - 2}$

19. $\dfrac{\hat{p} - p}{\sqrt{p(1-p)/n}} \doteq N(0, 1)$; $\hat{p} = \dfrac{x}{n}$

20. $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}} \doteq N(0, 1)$; $\hat{p} = \dfrac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$

21. $\underline{\hat{\beta}} = (X^T X)^{-1} X^T \mathbf{y} = X^+ \mathbf{y}$

22. $\hat{\sigma}^2 = \dfrac{1}{n-p}\|\mathbf{e}\|^2$; $\mathbf{e} = \mathbf{y} - X\underline{\hat{\beta}}$

23. $\tau_i = \dfrac{\hat{\beta}_i - \beta_i}{\hat{\sigma}\sqrt{c_{ii}}} \sim t(n-p)$; $c_{ii} = \left\{\left(X^T X\right)^{-1}\right\}_{ii}$

24. $\left[\ \hat{\beta}_i - t_1\hat{\sigma}\sqrt{c_{ii}}\ ,\ \hat{\beta}_i + t_1\hat{\sigma}\sqrt{c_{ii}}\ \right]$; $P(-t_1 \le \tau_i \le t_1) = 1 - \alpha$

N(0, 1)-distribution: $\Phi(z) = P(x \le z) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\,dt$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

t-distribution: $F(x) = P(t \le x)$; $t = \dfrac{u}{\sqrt{w/\gamma}}$, $u \sim N(0, 1)$, $w \sim \chi^2(\gamma)$

γ \ F	0.60	0.75	0.90	0.95	0.975	0.99	0.995	0.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.291