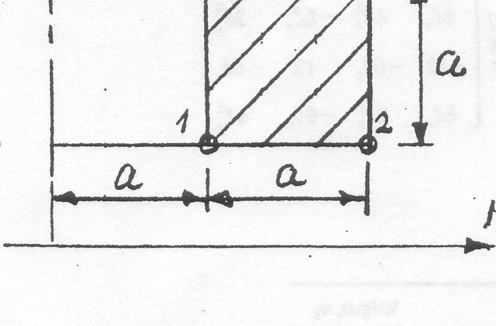
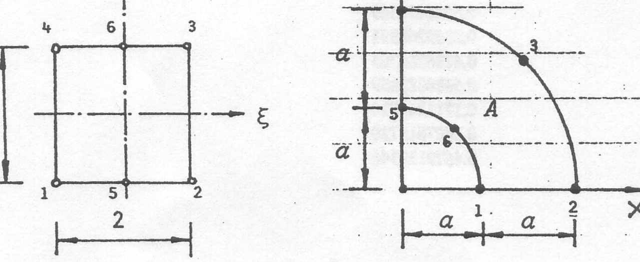
TTY/Teknillinen mekaniikka ja optimointi
23591 ELEMENTTIMENETELMÄN PERUSTEET

Välikoe 2 /2003S 3.12.2003



1. Määritä oheisen neliöpoikkileikkauksisen renkaselementin ekvivalenttiset solmuvoimat, kun kuormituksena on z-akselin negatiiviseen suuntaan vaikuttava maan vetovoima, jonka tiheys ρg on vakio.



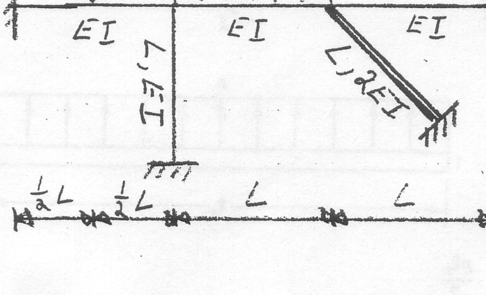
2. Oheinen kuusisolmuinen nelikulmioelementti, jonka muotofunktiot $N_i(\xi,\eta)$ $i=1,\ldots,6$ ovat ohessa on kuvattu xy-tasolle. Määritä kuvauksen Jacobin matriisi $[J]$ sekä $\det[J]$.

$$N_1 = -\frac{1}{4}\xi(1-\xi)(1-\eta) \qquad N_2 = \frac{1}{4}\xi(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}\xi(1+\xi)(1+\eta) \qquad N_4 = -\frac{1}{4}\xi(1-\xi)(1+\eta)$$

$$N_5 = \frac{1}{2}(1-\xi^2)(1-\eta) \qquad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$

3. Laske Gaussin yhden, kahden ja kolmen pisteen integrointia käyttäen arvo integraalille $I = \int_0^4 e^{-3x}dx$. Vertaa tulosta tarkkaan arvoon.



4. Määritä elementtimenetelmällä oheisen kehän taivutusmomenttikuvio, kun tasainen kuormitus $q = \dfrac{2F}{L}$. Palkit ovat venymättömiä.

KÄÄNNÄ

$$\mathbf{B} = \frac{1}{\det \mathbf{J}}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$dx\,dy = \det \mathbf{J}\, d\xi\, d\eta$$

$$\mathbf{k}^e = t_e A_e \mathbf{B}^T \mathbf{D} \mathbf{B}$$

$$\mathbf{k}^e = t_e \cdot \int_{-1}^{1}\int_{-1}^{1} \mathbf{B}^T \mathbf{D} \mathbf{B} \det \mathbf{J}\, d\xi\, d\eta$$

$$\mathbf{k}^e = \frac{EI}{\ell_e^3}\begin{bmatrix} 12 & 6\ell_e & -12 & 6\ell_e \\ 6\ell_e & 4\ell_e^2 & -6\ell_e & 2\ell_e^2 \\ -12 & -6\ell_e & 12 & -6\ell_e \\ 6\ell_e & 2\ell_e^2 & -6\ell_e & 4\ell_e^2 \end{bmatrix}$$

$$\int_{-1}^{1} f(\xi)\, d\xi \approx \sum_{i=1}^{n} w_i f(\xi_i)$$

Number of points, n	Location, ξ_i	Weights, w_i
1	0.0	2.0
2	$\pm 1/\sqrt{3} = \pm 0.5773502692$	1.0
3	$\pm$ 0.7745966692	0.5555555556
	0.0	0.8888888889
4	$\pm$ 0.8611363116	0.3478548451
	$\pm$ 0.3399810436	0.6521451549
5	$\pm$ 0.9061798459	0.2369268851
	$\pm$ 0.5384693101	0.4786286705
	0.0	0.5688888889
6	$\pm$ 0.9324695142	0.1713244924
	$\pm$ 0.6612093865	0.3607615730
	$\pm$ 0.2386191861	0.4679139346

$$\{\hat{r}\} = \iiint_{V^{(e)}} [N]^T \{f\}\, dV + \iint_{S^{(e)}} [N]^T \{p\}\, dS$$

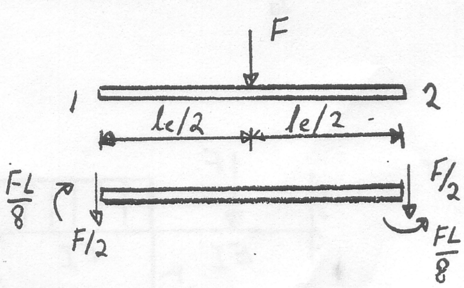
$$\mathbf{m}^e = \rho \int_e \mathbf{N}^T \mathbf{N}\, dV$$

$$N_i = \tfrac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i)$$

$$N_1 = \tfrac{1}{4}(1 - \xi)(1 - \eta)$$
$$N_2 = \tfrac{1}{4}(1 + \xi)(1 - \eta)$$
$$N_3 = \tfrac{1}{4}(1 + \xi)(1 + \eta)$$
$$N_4 = \tfrac{1}{4}(1 - \xi)(1 + \eta)$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{420}\begin{bmatrix} 156 & 22\ell_e & 54 & -13\ell_e \\ & 4\ell_e^2 & 13\ell_e & -3\ell_e^2 \\ & \text{Symmetric} & 156 & -22\ell_e \\ & & & 4\ell_e^2 \end{bmatrix}$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 \\ & & 1 & 0 \\ \text{Symmetric} & & & 1 \end{bmatrix} \qquad \mathbf{m}^e = \frac{\rho A_e \ell_e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 1 & 0 \\ \text{Symmetric} & & & 0 \end{bmatrix}$$



$$\det(\mathbf{K} - \lambda \mathbf{M}) = 0 \qquad \mathbf{K}\mathbf{U}_i = \lambda_i \mathbf{M}\mathbf{U}_i$$

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} \end{Bmatrix} = \mathbf{J}\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} \qquad \mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}$$

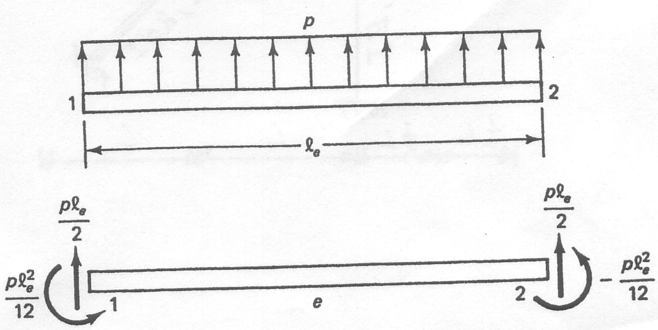


Figure 8.6 Distributed load on an element.