23591 ELEMENTTIMENETELMÄN PERUSTEET

Tentti 11.3.2002

Kirjallisuuden ja muistiinpanojen esilläpito <u>ei ole</u> sallittua. Jokaiseen vastauspaperiin on kirjoitettava nimi, nimenselvennös, opiskelijanumero, osasto ja vuosikurssi.

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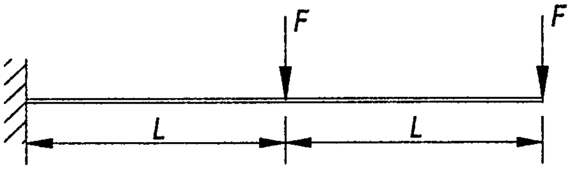
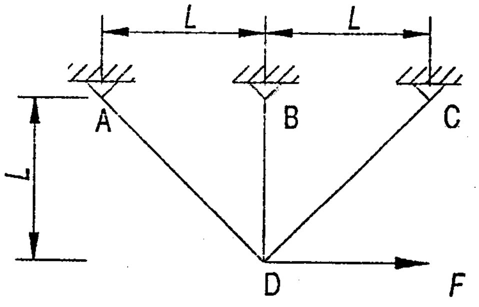
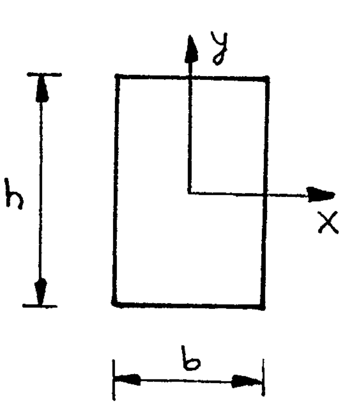


1. Laske Rayleigh-Ritzin menetelmällä kuvassa olevan palkin vapaan pään taipuma. Käytä kahta kinemaattisesti käypää kantafunktiota. Tutki, toteutuvatko palkin päiden reunaehdot. Piirrä palkin vapaakappalekuva ja katso siitä, onko palkki tasapainossa. Tehtävässä EI on vakio.



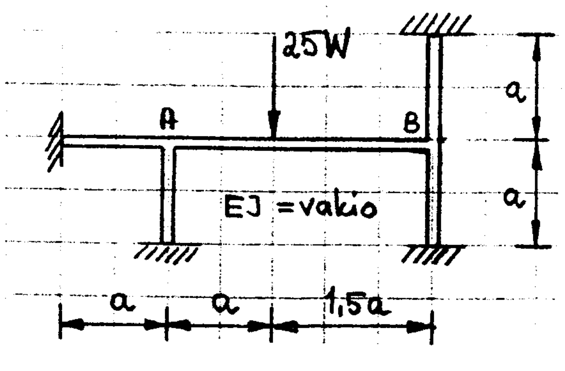
2. Kuvan ristikon niveleen D vaikuttaa vaakasuuntainen voima $F = 5$ kN. Ristikon kaikilla sauvoilla on sama poikkipinta-ala $A = 100$ mm^2 ja ne ovat terästä, jonka $E = 207$ GPa ja $\alpha = 12\ \mu/{}^{\circ}\mathrm{C}$. Laske nivelen D siirtymät ja sauvassa BD oleva normaalijännitys, kun pituusmitta $L = 1$ m.



3. Esitä oheisen suorakulmion Jacobin matriisi ja laske Gaussin numeerisella integroinnilla

   a) pinta-ala $A = \int\int \mathrm{d}A$

   b) neliömomentti $I_x = \int\int y^2\, \mathrm{d}A$.



4. Oheisen tasokehän palkit oletetaan venymättömiksi. Ratkaise kehän statiikka elementtimenetelmällä käyttäen vapausasteina nurkkien kiertymiä. Piirrä lisäksi kehän taivutusmomenttijakautuma.

TTKK/Koneosasto
Teknillinen mekaniikka ja optimointi

23591 Elementtimenetelmän perusteet syksy 2001

Kaavakokoelma

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + f_x = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + f_y = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0$$

$$\sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z = T_x$$
$$\tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z = T_y$$
$$\tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z = T_z$$

$$\epsilon = \left[\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}, \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^T$$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$
$$\epsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$
$$\epsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$
$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma = \mathbf{D}(\epsilon - \epsilon_0)$$

$$U = \frac{1}{2}\int_V \sigma^T \epsilon \, dV$$

$$\epsilon_0 = [\alpha\Delta T, \alpha\Delta T, \alpha\Delta T, 0, 0, 0]^T$$

$$\sigma = \mathbf{D}\epsilon$$

$$WP = -\int_V \mathbf{u}^T\mathbf{f}\, dV - \int_S \mathbf{u}^T\mathbf{T}\, dS - \sum_i \mathbf{u}_i^T\mathbf{P}_i$$

$$\int_V \sigma^T\epsilon(\phi)\, dV - \int_V \phi^T\mathbf{f}\, dV - \int_S \phi^T\mathbf{T}\, dS - \sum_i \phi^T\mathbf{P} = 0$$

$$\xi = \frac{2}{x_2 - x_1}(x - x_1) - 1$$

$$N_1(\xi) = \frac{1-\xi}{2}$$

$$N_2(\xi) = \frac{1+\xi}{2}$$

$$\mathbf{k}^e = \frac{E_e A_e}{\ell_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{f}^e = \frac{A_e \ell_e f}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \qquad \mathbf{T}^e = \frac{T\ell_e}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \qquad \Theta^e = \frac{E_e A_e \ell_e \alpha \Delta T}{x_2 - x_1}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$\mathbf{k} = \frac{E_e A_e}{\ell_e}\begin{bmatrix} \ell^2 & \ell m & -\ell^2 & -\ell m \\ \ell m & m^2 & -\ell m & -m^2 \\ -\ell^2 & -\ell m & \ell^2 & \ell m \\ -\ell m & -m^2 & \ell m & m^2 \end{bmatrix}$$

$$\sigma = \frac{E_e}{\ell_e}[-\ell \quad -m \quad \ell \quad m]\mathbf{q}$$

$$U_e = \tfrac{1}{2}\mathbf{q}^T\mathbf{k}\mathbf{q}$$

$$\mathbf{k} = \frac{E_e A_e}{\ell_e}\begin{bmatrix} \ell^2 & \ell m & \ell n & -\ell^2 & -\ell m & -\ell n \\ \ell m & m^2 & mn & -\ell m & -m^2 & -mn \\ \ell n & mn & n^2 & -\ell n & -mn & -n^2 \\ -\ell^2 & -\ell m & -\ell n & \ell^2 & \ell m & \ell n \\ -\ell m & -m^2 & -mn & \ell m & m^2 & mn \\ -\ell n & -mn & -n^2 & \ell n & mn & n^2 \end{bmatrix}$$

$$\ell = \frac{x_2 - x_1}{\ell_e} \qquad m = \frac{y_2 - y_1}{\ell_e} \qquad n = \frac{z_2 - z_1}{\ell_e}$$

$$\ell_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\mathbf{B} = \frac{1}{\det \mathbf{J}}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$dx\,dy = \det \mathbf{J}\,d\xi\,d\eta$$

$$\mathbf{k}^e = t_e A_e \mathbf{B}^T \mathbf{D} \mathbf{B}$$

$$\mathbf{k}^e = \frac{EI}{\ell_e^3}\begin{bmatrix} 12 & 6\ell_e & -12 & 6\ell_e \\ 6\ell_e & 4\ell_e^2 & -6\ell_e & 2\ell_e^2 \\ -12 & -6\ell_e & 12 & -6\ell_e \\ 6\ell_e & 2\ell_e^2 & -6\ell_e & 4\ell_e^2 \end{bmatrix}$$

$$\mathbf{k}^e = t_e \int_{-1}^{1}\int_{-1}^{1} \mathbf{B}^T\mathbf{D}\mathbf{B} \det \mathbf{J}\,d\xi\,d\eta$$

$$\int_{-1}^{1} f(\xi)\,d\xi \approx \sum_{i=1}^{n} w_i f(\xi_i)$$

Number of points, n	Location, ξ_i	Weights, w_i
1	0.0	2.0
2	$\pm 1/\sqrt{3} = \pm 0.5773502692$	1.0
3	$\pm$ 0.7745966692	0.5555555556
	0.0	0.8888888889
4	$\pm$ 0.8611363116	0.3478548451
	$\pm$ 0.3399810436	0.6521451549
5	$\pm$ 0.9061798459	0.2369268851
	$\pm$ 0.5384693101	0.4786286705
	0.0	0.5688888889
6	$\pm$ 0.9324695142	0.1713244924
	$\pm$ 0.6612093865	0.3607615730
	$\pm$ 0.2386191861	0.4679139346

$$\{\hat{f}\} = \iiint_{V^{(e)}} [N]^T\{f\}\,dV + \iint_{S^{(e)}} [N]^T\{p\}\,dS$$

$$\mathbf{m}^e = \rho \int_e \mathbf{N}^T\mathbf{N}\,dV$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{420}\begin{bmatrix} 156 & 22\ell_e & 54 & -13\ell_e \\ & 4\ell_e^2 & 13\ell_e & -3\ell_e^2 \\ & \text{Symmetric} & 156 & -22\ell_e \\ & & & 4\ell_e^2 \end{bmatrix}$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ & 1 & 0 & 0 \\ & & 1 & 0 \\ \text{Symmetric} & & & 1 \end{bmatrix}$$

$$\mathbf{m}^e = \frac{\rho A_e \ell_e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 1 & 0 \\ \text{Symmetric} & & & 0 \end{bmatrix}$$

$$\det(\mathbf{K} - \lambda \mathbf{M}) = 0 \qquad \mathbf{K}\mathbf{U}_i = \lambda_i \mathbf{M}\mathbf{U}_i$$

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} \end{Bmatrix} = \mathbf{J}\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}$$

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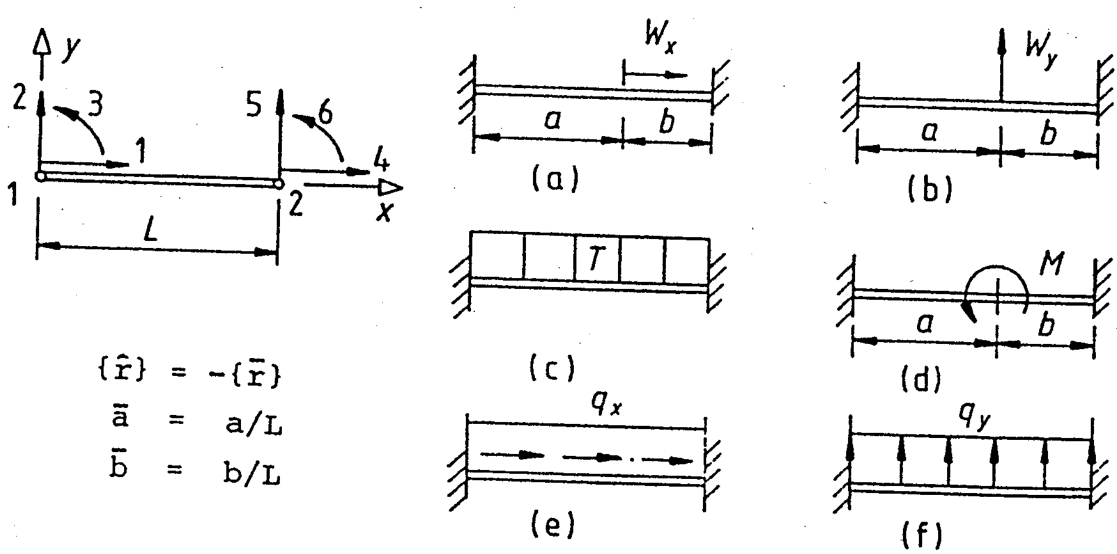
UDK 624.04:681.3
539.4.01

LUJUUSOPIN ELEMENTTIMENETELMÄN KÄYTTÖ

Nide 1



Kuorm. tapaus	Ekvivalenttisen solmukuormituksen komponentit f_i					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
a	$W_x\bar{b}$	0	0	$W_x\bar{a}$	0	
b	0	$W_y(1+2\bar{a})\bar{b}^2$	$W_y a\bar{b}^2$	0	$W_y(1+2\bar{b})\bar{a}^2$	$-W_y b\bar{a}^2$
c	$-EA\alpha T$	0	0	$EA\alpha T$	0	0
d	0	$-M6\bar{a}\bar{b}/L$	$M(1-3\bar{a})\bar{b}$	0	$M6\bar{a}\bar{b}/L$	$M(1-3\bar{b})\bar{a}$
e	$q_xL/2$	0	0	$q_xL/2$	0	0
f	0	$q_yL/2$	$q_yL^2/12$	0	$q_yL/2$	$-q_yL^2/12$

Kuva 1. Ekvivalenttisia solmukuormituksia.