73040 Vektorianalyysin kaavoja (K,Te,Tu,M,R,Y,Tj)

1. $\displaystyle\iint_{R_{xy}} f(x,y)\,dx\,dy = \iint_{R_{uv}} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$

2. $\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases} \implies \dfrac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} = \rho^2\sin\phi$

3. $\displaystyle\int_C f\,ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|\,dt$

4. $\displaystyle\int_C \mathbf{F}\bullet d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\bullet\mathbf{r}'(t)\,dt = \int_C \mathbf{F}\bullet\mathbf{T}\,ds = \int_C f_1\,dx + \int_C f_2\,dy + \int_C f_3\,dz$

5. $\displaystyle\oint_{\partial R} \mathbf{F}\bullet d\mathbf{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$

6. $\displaystyle\oint_{\partial R} \mathbf{F}\bullet\mathbf{n}\,ds = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dx\,dy$

7. $\displaystyle\iint_S f\,d\sigma = \iint_R f(\mathbf{r}(u,v))\|\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v)\|\,du\,dv$

8. $\displaystyle\iint_S f\,d\sigma = \iint_R f(x,y,z(x,y))\sqrt{1+z_x^2+z_y^2}\,dx\,dy$

9. $\displaystyle\iint_S \mathbf{F}\bullet\mathbf{n}\,d\sigma = \iint_R \mathbf{F}(\mathbf{r}(u,v))\bullet\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v)\,du\,dv$

10. $\displaystyle\iint_S \mathbf{F}\bullet\mathbf{n}\,d\sigma = \iint_R \mathbf{F}(x,y,z(x,y))\bullet(-z_x,-z_y,1)\,dx\,dy$

11. (i) $\nabla\times\nabla h = 0$
    (ii) $\nabla\bullet(\nabla\times\mathbf{F}) = 0$
    (iii) $\nabla\bullet(h\mathbf{F}) = \nabla h\bullet\mathbf{F} + h\nabla\bullet\mathbf{F}$
    (iv) $\nabla\times(h\mathbf{F}) = \nabla h\times\mathbf{F} + h\nabla\times\mathbf{F}$
    (v) $\nabla(\mathbf{F}\bullet\mathbf{G}) = \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F}) + (\mathbf{F}\bullet\nabla)\mathbf{G} + (\mathbf{G}\bullet\nabla)\mathbf{F}$
    (vi) $\nabla\bullet(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\bullet(\nabla\times\mathbf{F}) - \mathbf{F}\bullet(\nabla\times\mathbf{G})$
    (vii) $\nabla\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\bullet\mathbf{G}) - (\mathbf{F}\bullet\nabla)\mathbf{G} + (\mathbf{G}\bullet\nabla)\mathbf{F} - \mathbf{G}(\nabla\bullet\mathbf{F})$
    (viii) $\nabla\times(\nabla\times\mathbf{F}) = \nabla(\nabla\bullet\mathbf{F}) - \nabla^2\mathbf{F}$

12. $\displaystyle\oiint_S \mathbf{F}\bullet\mathbf{n}\,d\sigma = \iiint_R \nabla\bullet\mathbf{F}\,dv$

13. $\displaystyle\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,d\sigma$

14. $\displaystyle A(\mathbf{r}) = \int_0^1 t\mathbf{F}(\mathbf{r}_0 + t(\mathbf{r}-\mathbf{r}_0))\times(\mathbf{r}-\mathbf{r}_0)\,dt$

15. $\displaystyle A(\mathbf{r}) = \left(\int_{z_0}^z f_2(x,y,z)\,dz - \int_{y_0}^y f_3(x,y,z_0)\,dy,\ -\int_{z_0}^z f_1(x,y,z)\,dz,\ 0\right)$

16. $\displaystyle u(\mathbf{r}) = \int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F}\bullet d\mathbf{r}$

$$\sin(2t) = 2\sin t\cos t \qquad \sin^2 t = \frac{1-\cos(2t)}{2} \qquad \cos^2 t = \frac{1+\cos(2t)}{2}$$