23615 Optimointi ja päätöksenteko
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Kirjallisuuden ja muistiinpanojen esilläpito on kielletty. Kirjoita jokaiseen vastauspaperiin selvästi opiskelijanumero, nimikirjoitus ja nimenselvennös.

1. The Kazak Film company needs to cut 15 long rolls and 10 short rolls of film from stock pieces. Each stock piece can be cut in one of two patterns. The first produces 5 long and 2 short rolls; the second yields 3 long and 5 short. Once any part of a piece of stock is cut, anything that remains is scrap. Also, neither pattern should be used more than 4 times because the jig used to cut it will become too inaccurate. Kazak wants to find the allowable combination of patterns that will minimize the number of stock pieces required.

Formulate a mathematical model to decide what patterns to use. Use decision variables $x_1 \triangleq$ number of times pattern 1 is used and $x_2 \triangleq$ number of times pattern 2 is used.

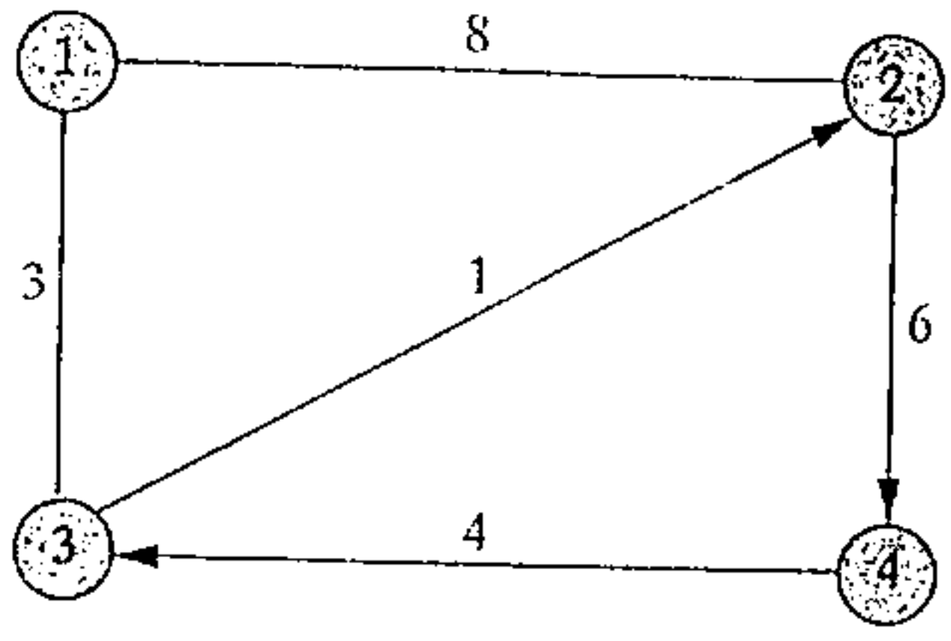
Solve the problem graphically.

2. Ratkaise LP-ongelma

$$\begin{aligned} \max z &= 3x_1 + x_2 \\ -2x_1 + x_2 &\le 2 \\ x_1 + x_2 &\le 6 \\ x_1 &\le 4 \\ x_1, x_2 &\ge 0 \end{aligned}$$

käyttäen simplex algoritmia.

3. Consider the following graph.



Käyttäen Floyd-Warshall-algoritmia etsi lyhimmät polut kaikista solmuista kaikkiin muihin solmuihin.

ALGORITHM 9B: ALL TO ALL (NO NEGATIVE DICYCLES); FLOYD–WARSHALL

Step 0: Initialization. All nodes should have consecutive positive numbers starting with 1. For all arcs and edges (k, ℓ) in the graph, initialize

$$v^{(0)}[k, \ell] \leftarrow c_{k,\ell}$$
$$d[k, \ell] \leftarrow k$$

For k, ℓ pairs with no arc/edge (k, ℓ), assign

$$v^{(0)}[k, \ell] \leftarrow \begin{cases} 0 & \text{if } k = \ell \\ +\infty & \text{otherwise} \end{cases}$$

Also set iteration counter $t \leftarrow 1$.

Step 1: Evaluation. For all $k, \ell \neq t$ update

$$v^{(t)}[k, \ell] \leftarrow \min\{v^{(t-1)}[k, \ell],\ v^{(t-1)}[k, t] + v^{(t-1)}[t, \ell]\}$$

If $v^{(t)}[k, \ell] < v^{(t-1)}[k, \ell]$, also set $d[k, \ell] \leftarrow d[t, \ell]$.

Step 2: Stopping. Terminate if $t =$ the number of nodes in the graph, or if $v^{(t)}[k, k] < 0$ for any node k. Values $v^{(t)}[k, \ell]$ then equal the required shortest path lengths unless some $v^{(t)}[k, k]$ is negative, in which case the graph contains a negative dicycle through k.

Step 3: Advance. If $t <$ the number of nodes and all $v^{(t)}[k, k] \geq 0$, increment $t \leftarrow t + 1$ and return to Step 1.

4. Ota gradienttimenetelmää käyttäen yksi optimointiaskel lähtöpisteestä $\underline{x}^0 = (30,2)$, kun maksimoitavana on funktio

$$f(x_1,x_2) = \frac{x_1}{1+e^{x_1/10}} - (x_2-5)^2$$

Optimointiaskel sisältää suunnan valinnan sekä suoralta haun, johon tulee soveltaa kultaisen leikkauksen menetelmää.

5. Tutki Pareto-optimin määritelmään perustuen, mikä on seuraavan kaksikriteerisen ongelman

$$\max_{0\le x\le 1} \begin{bmatrix} -(x-0.3)^2 \\ -(x-0.7)^2 \end{bmatrix}$$

Pareto-optimaalinen joukko suunnitteluavaruudessa.