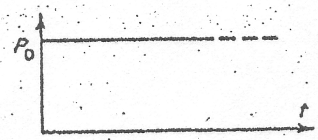
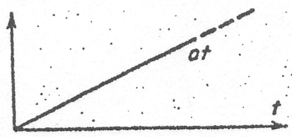
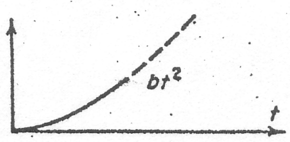
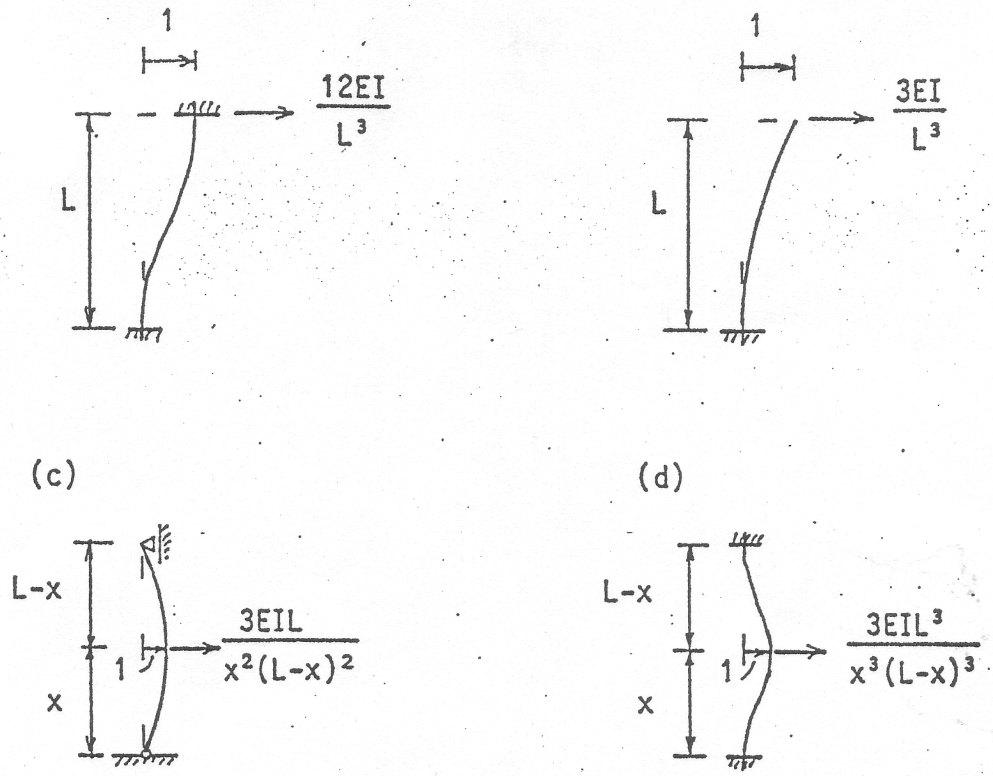
TTKK/K TME
23140 Rakenteiden dynamiikka

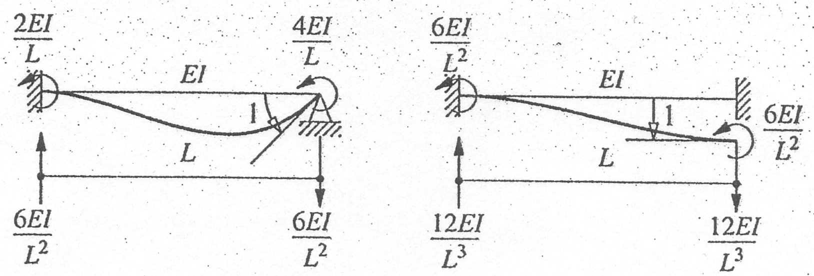
Duhamelin integraaleja

Tapaus n:o	Voimafunktio P(t)	$\frac{1}{m\omega}\int_{t_1}^{t_2} P(\tau)\sin[\omega(t-\tau)]d\tau$
1.		$\frac{P_0}{m\omega}\Big\vert_{t_1}^{t_2}\frac{1}{\omega}\cos[\omega(t-\tau)]$
2.		$\frac{1}{m\omega}\left\{\Big\vert_{t_1}^{t_2}\frac{1}{m\omega}a\tau\cos[\omega(t-\tau)] + \frac{a}{\omega}\Big\vert_{t_1}^{t_2}\frac{1}{\omega}\sin[\omega(t-\tau)]\right\}$
3.		$\frac{1}{m\omega}\left\{\frac{b}{\omega}\Big\vert_{t_1}^{t_2}\tau^2\cos[\omega(t-\tau)] - \frac{2b}{\omega^2}\Big\vert_{t_1}^{t_2}\tau\sin[\omega(t-\tau)] - \frac{2b}{\omega^3}\Big\vert_{t_1}^{t_2}\cos[\omega(t-\tau)]\right\}$

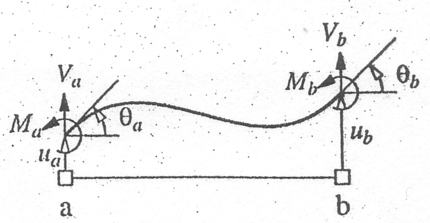
F = k x

Sauvojen jäykkyyksiä

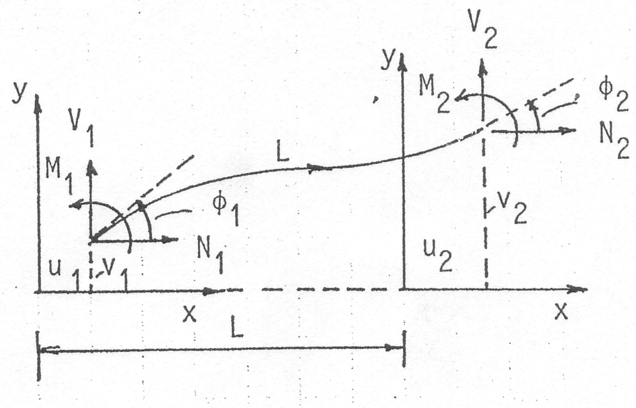




(a) (b)



(c)



$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \varphi_1 \end{Bmatrix} + \begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ \varphi 2 \end{Bmatrix}$$

$$\begin{Bmatrix} N_2 \\ V_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \varphi_1 \end{Bmatrix} + \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ \varphi_2 \end{Bmatrix}$$

eli

$$\bar{p}_1 = \bar{\bar{K}}_{11}\bar{d}_1 + \bar{\bar{K}}_{12}\bar{d}_2$$

$$\bar{p}_2 = \bar{\bar{K}}_{21}\bar{d}_1 + \bar{\bar{K}}_{22}\bar{d}_2$$

Kun sauvojen venymät jätetään huomiotta. jolloin solmusiirtymiin tulee sitovia ehtoja, saadaan (asettamalla molempien sauvanpäiden siirtymävektorit yhdeksi vektoriksi)

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}_E = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \bar{\bar{k}}\bar{d} \qquad \text{("E" kimmoisuudesta)}$$

Vastaavasti massamatriisi

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}_I = \frac{mL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \begin{Bmatrix} \ddot{v}_1 \\ \ddot{\varphi}_1 \\ \ddot{v}_2 \\ \ddot{\varphi}_2 \end{Bmatrix} = \bar{\bar{m}}\ddot{\bar{d}} \qquad \text{("I" hitaudesta)}$$