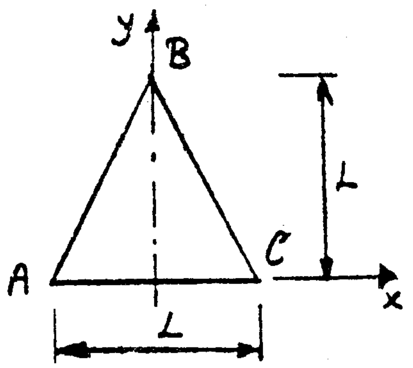
TME-2100 Materiaalien mekaniikka

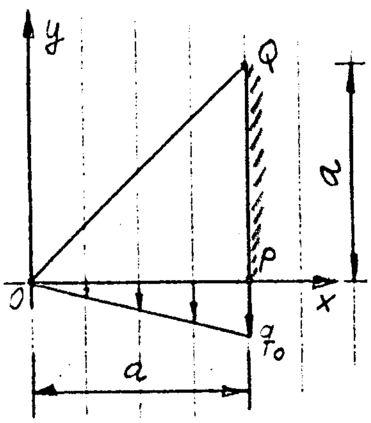
Tentti 31.1.2008

Kirjallisuutta ja muistiinpanoja ei saa pitää esillä. Kaavakokoelma on palautettava.

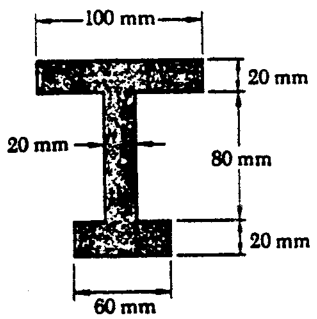
1. Rakenteen erään pisteen jännitystilan jännityskomponentit ovat $\sigma_{xx} = -40$ MPa, $\sigma_{yy} = 80$ MPa, $\sigma_{zz} = 120$ MPa, $\tau_{xy} = 72$ MPa, $\tau_{yz} = 46$ MPa ja $\tau_{xz} = 32$ MPa. Laske normaali- sekä leikkausjännitys tasossa, jonka normaali muodostaa kulman 48^0 x-akselin ja kulman 61^0 y-akselin kanssa.



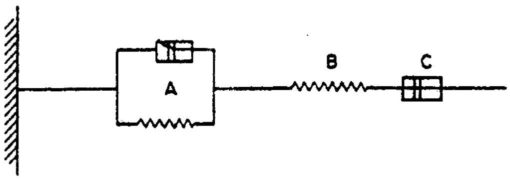
2. Teräslevyssä on homogeeninen tasojännitystilakenttä, jonka komponentit ovat $\sigma_x = 180$ MPa $\sigma_y = -100$ MPa ja $\tau_{xy} = 80{,}8$ MPa. Määritä levyn muodonmuutostilakenttä sekä kulman ABC muutos. $E = 210$ GPa, $\nu = 0{,}3$.



3. Oheisen kolmiolevyn OPQ sivun OP normaalijännityksen lauseke on $q_0 \dfrac{x}{a}$. Sivu OQ on kuormittamaton. Käytä jännitysfunktiota $\phi = Ax^3 + Bx^2y + Cxy^2 + Cy^3$. Määritä vakiot A, B, C ja D sekä sivun PQ jännitysten lausekkeet.



4. Laske oheisen poikkileikkauksen kantomomentti M_p, kun materiaali on kuparia, jonka myötöraja $R_e = 50$ MPa. Laske myös kantomomentin ja myötömomentin suhde $\dfrac{M_p}{M_m}$.



5. Oheista mekaanista mallia käytetään simuloimaan polymeerien käyttäytymistä. Sen konstitutiivinen yhtälö on

$$\eta_A \frac{d^2\varepsilon}{dt^2} + E_A \frac{d\varepsilon}{dt} = \frac{\eta_A}{E_B} \frac{d^2\sigma}{dt^2} + \left(1 + \frac{E_A}{E_B} + \frac{\eta_A}{\eta_C}\right) \frac{d\sigma}{dt} + \frac{E_A}{\eta_C} \sigma .$$

Polymeeria kuormitetaan vakiojännityksellä 6 kPa 30 sekunnin ajan. Määritä polymeerin venymä kuormituksen lopetushetkellä. $E_A = 50$ kPa, $E_B = 1$ GPa, $\eta_A = 10^6$ Ns/m^2 ja $\eta_C = 100 \cdot 10^6$ Ns/m^2.

TTY/Teknillinen mekaniikka ja optimointi

TME-2100 Materiaalien mekaniikka

PALAUTA, ÄLÄ HYLJÄ!

KAAVOJA

$$\sigma_x,_x+\tau_{xy},_y+\tau_{xz},_z+f_x=0$$
$$\tau_{xy},_x+\sigma_y,_y+\tau_{yz},_z+f_y=0 \qquad A_i=\begin{vmatrix}\varepsilon_y-\varepsilon_i & \varepsilon_{yz}\\ \varepsilon_{yz} & \varepsilon_z-\varepsilon_i\end{vmatrix} \quad B_i=-\begin{vmatrix}\varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yz} & \varepsilon_z-\varepsilon_i\end{vmatrix} \quad C_i=\begin{vmatrix}\varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{y}-\varepsilon_i & \varepsilon_{yz}\end{vmatrix}$$
$$\tau_{xz},_x+\tau_{yz},_y+\sigma_z,_z+f_z=0$$

$$\sigma_\alpha=\sigma_x l^2+\sigma_y m^2+\sigma_z n^2+2\tau_{xy}lm+2\tau_{yz}mn+2\tau_{xz}\ln \qquad \sigma_\alpha=\{e_\alpha\}\cdot[S]\{e_\alpha\} \qquad e=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$$

$$\varepsilon_{x'}=\varepsilon_x l^2+\varepsilon_y m^2+\varepsilon_z n^2+2\varepsilon_{xy}lm+2\varepsilon_{yz}mn+2\varepsilon_{xz}\ln \qquad \varepsilon=\{e\}^T[V]\{e\} \qquad \gamma_{xy}=2\varepsilon_{xy}$$

$$\varepsilon_{x'}=\frac{1}{2}(\varepsilon_x+\varepsilon_y)+\frac{1}{2}(\varepsilon_x-\varepsilon_y)\cos2\theta+\varepsilon_{xy}\sin2\theta \qquad \varepsilon_{x'y'}=-\frac{1}{2}(\varepsilon_x-\varepsilon_y)\sin2\theta+\varepsilon_{xy}\cos2\theta$$

$$R_i=\sqrt{A_i^2+B_i^2+C_i^2} \qquad \varepsilon^3-I_1\varepsilon^2+I_2\varepsilon-I_3=0 \qquad (1+\varepsilon)^2=(\{e\}+[D]\{e\})^2$$

$$l_i=\frac{A_i}{R_i},\quad m_i=\frac{B_i}{R_i},\quad n_i=\frac{C_i}{R_i} \qquad \{p_\alpha\}=[S]\{e_\alpha\} \qquad K=-\frac{p}{e}=\frac{E}{3(1-2\nu)} \qquad G=\frac{E}{2(1+\nu)}$$

$$J_1=\varepsilon_x+\varepsilon_y+\varepsilon_z \qquad J_2=\begin{vmatrix}\varepsilon_x & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_y\end{vmatrix}+\begin{vmatrix}\varepsilon_x & \varepsilon_{xz}\\ \varepsilon_{xz} & \varepsilon_z\end{vmatrix}+\begin{vmatrix}\varepsilon_y & \varepsilon_{yz}\\ \varepsilon_{yz} & \varepsilon_z\end{vmatrix} \qquad J_3=\det[V] \qquad [S]'=[Q]^T[S][Q]$$

$$\varepsilon_{1,2}=\frac{\varepsilon_x+\varepsilon_y}{2}\pm\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\varepsilon_{xy}^2} \qquad \tan2\varphi=\frac{2\varepsilon_{xy}}{\varepsilon_x-\varepsilon_y} \qquad \varepsilon_{xy}\sin2\varphi\ge0 \qquad \nabla^4\phi=\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=0$$

$$\begin{cases}\varepsilon_x,_{yy}+\varepsilon_y,_{xx}=\gamma_{xy},_{xy}\\ \varepsilon_y,_{zz}+\varepsilon_z,_{yy}=\gamma_{yz},_{yz}\\ \varepsilon_z,_{xx}+\varepsilon_x,_{zz}=\gamma_{zx},_{zx}\end{cases} \qquad \varepsilon_x=u,_x \quad \varepsilon_y=v,_y \quad \varepsilon_z=w,_z$$
$$\gamma_{xy}=u,_y+v,_x \qquad \gamma_{zx}=u,_z+w,_x \qquad \gamma_{yz}=v,_z+w,_y$$

$$\begin{cases}2\varepsilon_x,_{yz}=\frac{\partial}{\partial x}(-\gamma_{yz},_x+\gamma_{zx},_y+\gamma_{xy},_z)\\ 2\varepsilon_y,_{zx}=\frac{\partial}{\partial y}(\gamma_{yz},_x-\gamma_{zx},_y+\gamma_{xy},_z)\\ 2\varepsilon_z,_{xy}=\frac{\partial}{\partial z}(\gamma_{yz},_x+\gamma_{zx},_y-\gamma_{xy},_z)\end{cases} \qquad \sigma_x=\frac{\partial^2\phi}{\partial y^2} \qquad \sigma_y=\frac{\partial^2\phi}{\partial x^2} \qquad \tau_{xy}=-\frac{\partial^2\phi}{\partial x\partial y}$$

$$U_0=\int_0^{\varepsilon_x}\sigma_x\,d\varepsilon_x \qquad U_0^*=\int_0^{\sigma_x}\varepsilon_x\,d\sigma_x \qquad U_0=\tfrac{1}{2}(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_z\varepsilon_z+\tau_{xy}\gamma_{xy}+\tau_{yz}\gamma_{yz}+\tau_{xz}\gamma_{xz})$$

$$\varepsilon_{xx}=\frac{1}{E}[\sigma_{xx}-\nu(\sigma_{yy}+\sigma_{zz})] \qquad \sigma_{xx}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{xx}+\nu(\varepsilon_{yy}+\varepsilon_{zz})]$$

$$\varepsilon_{yy}=\frac{1}{E}[\sigma_{yy}-\nu(\sigma_{xx}+\sigma_{zz})] \qquad \sigma_{yy}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{yy}+\nu(\varepsilon_{xx}+\varepsilon_{zz})]$$

$$\varepsilon_{zz}=\frac{1}{E}[\sigma_{zz}-\nu(\sigma_{xx}+\sigma_{yy})] \qquad \sigma_{zz}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{zz}+\nu(\varepsilon_{xx}+\varepsilon_{yy})]$$

$$\gamma_{xy}=\frac{1}{G}\tau_{xy} \qquad \gamma_{yz}=\frac{1}{G}\tau_{yz} \qquad \gamma_{xz}=\frac{1}{G}\tau_{xz} \qquad \tau_{xy}=G\gamma_{xy} \qquad \tau_{yz}=G\gamma_{yz} \qquad \tau_{xz}=G\gamma_{xz}$$

suorakulmio: $M_m=\dfrac{bh^2}{6}$ $\qquad M_p=\dfrac{bh^2}{4}$ $\qquad M_{xy}=-D(1-\nu)\dfrac{\partial^2 w}{\partial x\partial y}$ $\qquad \varepsilon^3-I_1\varepsilon^2+I_2\varepsilon-I_3=0$

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y=-D\left(\nu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \qquad D=\frac{Eh^3}{12(1-\nu^2)}$$

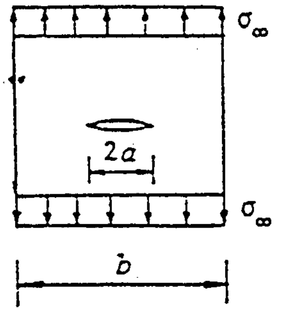
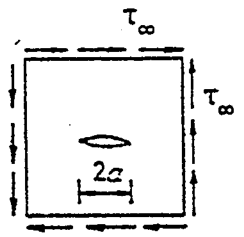
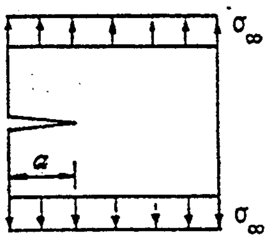
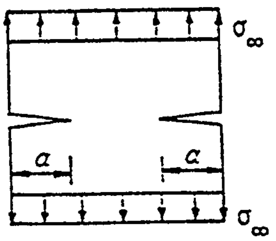
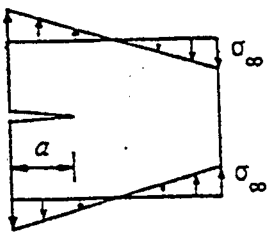
$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}=Q_x \qquad \frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}=Q_y \qquad \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=-p$$

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}=-p \qquad \frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{p(x,y)}{D}$$

$$Q_x=-D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \qquad Q_y=-D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

$$V_x(y)=Q_x(c_1,y)+\frac{\partial M_{xy}}{\partial y}(c_1,y) \qquad V_y(x)=Q_y(x,c_2)+\frac{\partial M_{xy}}{\partial x}(x,c_2)$$

Taulukko 1 Jännitysintensiteettikertoimia

1		$K_I = \sigma_\infty \sqrt{\pi a}\,\dfrac{1-(a/b)+1{,}304\,(a/b)^2}{\sqrt{1-2a/b}}$ $K_I = \sigma_\infty \sqrt{\pi a}$
2		$K_{II} = \tau_\infty \sqrt{\pi a}$
3		$a/b < 0{,}7$ $K_I = \sigma_\infty \sqrt{\pi a}\,(1{,}12-0{,}23(a/b)+10{,}6(a/b)^2+$ $-21{,}7(a/b)^3+30{,}4(a/b)^4)$ $K_I \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$
4		$K_I = \sigma_\infty \sqrt{\pi a}\,\dfrac{1{,}12-1{,}22(a/b)+1{,}04(a/b)^3}{\sqrt{1-2a/b}}$ $K_I \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$
5		$a/b < 0{,}7$ $K_I = \sigma_\infty \sqrt{\pi a}\,(1{,}12-1{,}39(a/b)+7{,}3(a/b)^2+$ $-13(a/b)^3+14(a/b)^4)$ $K_I \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$